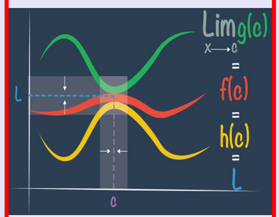
# Calculus I

# Lecture 18



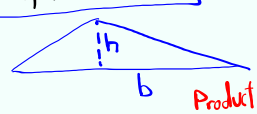

Feb 19-8:47 AM

The height of a triangle is increasing at a rate of 2 cm/min $\frac{dh}{dt}=2$

Its area is increasing at the rate of 10 $cm^2$/min $\frac{dA}{dt}=10$ ✓

At what rate is its base changing when its $\boxed{\text{height is 10 cm}}$ and its $\boxed{\text{area is 100 } cm^2}$?

$A=\frac{bh}{2}$ $\frac{db}{dt}=?$




Product $2A=bh$ $h=10$

$2(100)=b(10)$ $A=100$

$b=20$

$\frac{d}{dt}[2A]=\frac{d}{dt}[bh]$

$2\cdot 10=\frac{db}{dt}\cdot 10+20\cdot 2$

$2\frac{dA}{dt}=\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}$

$20=10\frac{db}{dt}+40$

$10\frac{db}{dt}=-20$

base is decreasing at 2cm/min.

$\frac{db}{dt}=-2$ cm/min

Apr 23-8:52 AM

An object is moving along the curve

$y = 2\sin\frac{\pi x}{2}$

As the object passes the point $(\overset{x}{\frac{1}{3}}, 1)$,

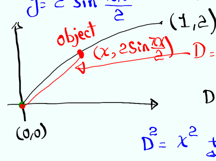
its x-Coordinate increases at the rate of

$\sqrt{10}$ cm/s. $\frac{dx}{dt} = \sqrt{10}$

How fast is the distance between the object

and the origin changint at that point? $\frac{dD}{dt} = ?$

$y = 2\sin\frac{\pi x}{2}$ (0,0) (1,2)




$D = \sqrt{(x-0)^2 + (2\sin\frac{\pi x}{2} - 0)^2}$

$D = \sqrt{x^2 + 4\sin^2\frac{\pi x}{2}}$

$D = \sqrt{\frac{1}{9} + 4\cdot\frac{1}{4}}$

$= \sqrt{\frac{1}{9} + 1}$

$= \frac{\sqrt{10}}{3}$

$D^2 = x^2 + 4\sin^2\frac{\pi x}{2}$

$\frac{d}{dt}[D^2] = \frac{d}{dt}[x^2 + 4\sin^2\frac{\pi x}{2}]$

$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 4\cdot 2\cdot\sin\frac{\pi x}{2}\cdot\cos\frac{\pi x}{2}\cdot\frac{\pi}{2}\cdot\frac{dx}{dt}$

$D\frac{dD}{dt} = x\frac{dx}{dt} + \boxed{2}\pi\boxed{\sin\frac{\pi x}{2}}\boxed{\cos\frac{\pi x}{2}}\cdot\frac{dx}{dt}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\to \sin\pi x$

$\frac{\sqrt{10}}{3}\cdot\frac{dD}{dt} = \frac{1}{3}\cdot\sqrt{10} + \pi\sin\pi\cdot\frac{1}{3}\cdot\sqrt{10}$

$\frac{\sqrt{10}}{3}\cdot\frac{dD}{dt} = \frac{\sqrt{10}}{3} + \pi\cdot\frac{\sqrt{3}}{2}\cdot\sqrt{10}$

$\frac{1}{3}\frac{dD}{dt} = \frac{1}{3} + \frac{\pi\sqrt{3}}{2}$

Multiply by 3

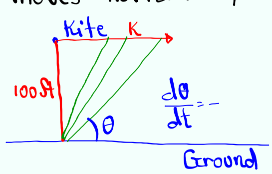
$\frac{dD}{dt} = 1 + \frac{3\pi\sqrt{3}}{2}$ cm/s.

Apr 23-9:03 AM

A Kite is 100 ft above the ground,

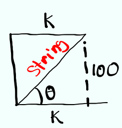
moves horizontally at the speed of 8ft/s.

$\frac{dK}{dt} = 8$



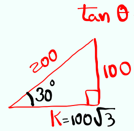

$\frac{d\theta}{dt} = -$

At what rate is the angle between the String and the ground changing when the string is 200ft out?




$\tan\theta = \frac{100}{K}$

$\tan\theta = 100K^{-1}$




$\sin\theta = \frac{100}{200}$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$\sec^2\theta\cdot\frac{d\theta}{dt} = 100\cdot -1\cdot K^{-2}\cdot\frac{dK}{dt}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{-100}{K^2}\cdot\frac{dK}{dt}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sec 30° = \frac{2}{\sqrt{3}}$

$\sec^2 30° = \frac{4}{3}$

$\frac{4}{3}\cdot\frac{d\theta}{dt} = \frac{-100}{(100\sqrt{3})^2}\cdot 8$

$\frac{4}{3}\frac{d\theta}{dt} = \frac{-\cancel{100}\cdot 8^{2}}{\cancel{100}\cdot 100\cdot 3}$

$\frac{d\theta}{dt} = \frac{-2}{100} = -.02$ Rad/S.

Apr 23-9:25 AM

$f(x) = x^3 - 12x^2 + 36x$

Find all points on the graph of $f(x)$ where $f'(x)$ and $f''(x)$ are zero or undefined.

$f'(x) = 3x^2 - 24x + 36$

$f''(x) = 6x - 24$

Polynomials Defined everywhere

$f'(x) = 0 \qquad 3x^2 - 24x + 36 = 0 \qquad x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6 \;,\; x = 2$

Points $(6, f(6)), (2, f(2))$

$f''(x) = 0 \qquad 6x - 24 = 0$

$x - 4 = 0$

$x = 4 \longrightarrow$ Point $(4, f(4))$

Apr 23-9:41 AM

$f(x) = \dfrac{x}{x^2+9}$ Domain $(-\infty, \infty)$

Find all points on the graph of $f(x)$ where $f'(x)$ and $f''(x)$ are zero or undefined.

$$f'(x) = \frac{1(x^2+9) - x \cdot 2x}{(x^2+9)^2} = \frac{9 - x^2}{(x^2+9)^2}$$

$f'(x) = 0 \qquad \dfrac{9-x^2}{(x^2+9)^2} = 0 \qquad 9 - x^2 = 0$

$x = \pm 3$

Points $(3, f(3)), (-3, f(-3))$

$$f''(x) = \frac{-2x(x^2+9)^2 - (9-x^2) \cdot 2(x^2+9) \cdot 2x}{\left[(x^2+9)^2\right]^2}$$

$$= \frac{-2x(x^2+9)^2 - 4x(9-x^2)(x^2+9)}{(x^2+9)^4}$$

$x^2+9+18-2x^2$

$$= \frac{-2x(x^2+9) - 4x(9-x^2)}{(x^2+9)^3} = \frac{-2x[x^2+9+2(9-x^2)]}{(x^2+9)^3}$$

$$= \frac{-2x[27-x^2]}{(x^2+9)^3}$$

$$= \frac{2x(x^2-27)}{(x^2+9)^3}$$

$f''(x) = 0$

$2x(x^2-27) = 0$

$x = 0 \qquad x = \pm\sqrt{27}$

$x = \pm 3\sqrt{3}$

Points $(0, f(0)), (3\sqrt{3}, f(3\sqrt{3})), (-3\sqrt{3}, f(-3\sqrt{3}))$

Apr 23-9:41 AM

Find all points on the graph of $f(x) = \sqrt[3]{x^5} - 5\sqrt[3]{x^2}$ where $f'(x)$ is Zero or undefined.

because of cube root $\Rightarrow$ Domain $(-\infty, \infty)$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$$f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$$

$$f'(x) = \frac{5}{3} x^{\frac{5}{3}-1} - 5 \cdot \frac{2}{3} x^{\frac{2}{3}-1}$$

$$= \frac{5}{3} \boxed{x^{\frac{2}{3}}} - \frac{10}{3} \boxed{x^{-\frac{1}{3}}}$$

$$= \frac{5}{3} x^{-1/3} \left[ x^{\boxed{1}} - 2 \right]$$

$$= \frac{5(x-2)}{3\sqrt[3]{x}}$$

$x^m \cdot x^n = x^{m+n}$

$x^{-1/3} \cdot x^1 = x^{-1/3+1} = x^{\frac{2}{3}}$

$x^{\frac{1}{3}} = \sqrt[3]{x^1} = \sqrt[3]{x}$

Point

$f'(x) = 0 \rightarrow 5(x-2) = 0 \rightarrow x = 2 \rightarrow (2, f(2))$

$f'(x)$ undefined $\rightarrow 3\sqrt[3]{x} = 0 \rightarrow x = 0 \rightarrow (0, f(0))$

Apr 23-10:07 AM

Find all points on the graph of $f(x) = x^3 - 12x + 2$ where $f'(x)$ & $f''(x)$ are Zero or undefined.

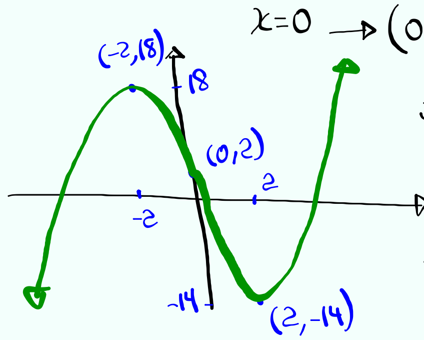
$f'(x) = 3x^2 - 12$ $\quad$ $f'(x) = 0$ $\quad$ $3x^2 - 12 = 0$

$x^2 - 4 = 0$

$f''(x) = 6x$ $\quad$ $f''(x) = 0$ $\quad$ $x = 2 \rightarrow (2, -14)$

$6x = 0$ $\quad$ $x = -2 \rightarrow (-2, 18)$

$x = 0 \rightarrow (0, 2)$




$f'(x) \rightarrow$ Increasing/Decreasing

$f''(x) \rightarrow$ Concavity

Apr 23-10:21 AM

Find all points on the graph of $f(x) = (x+1)^5 - 5x - 2$ where $f'(x)$ & $f''(x)$ are zero or undefined.

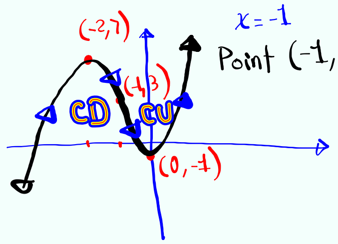
$f'(x) = 5(x+1)^4 \cdot 1 - 5$ $\qquad f'(x) = 5(x+1)^4 - 5$

$f''(x) = 5 \cdot 4(x+1)^3 \cdot 1 - 0$ $\qquad f''(x) = 20(x+1)^3$

$f'(x) = 0 \qquad 5(x+1)^4 - 5 = 0$
$\qquad 5(x+1)^4 = 5 \rightarrow (x+1)^4 = 1$
$\qquad x+1 = \pm\sqrt[4]{1}$
$\qquad x+1 = 1 \rightarrow x = 0$
$\qquad x+1 = -1 \rightarrow x = -2$
Points $(0,-1)$
$(-2,7)$

$f''(x) = 0 \qquad 20(x+1)^3 = 0$
$(x+1)^3 = 0$
$x+1 = 0$
$x = -1$

Point $(-1,3)$




Apr 23-10:35 AM

Find a function that satisfies the following
$\boxed{\text{Two numbers}}$ with difference of 100
Function describes their product.

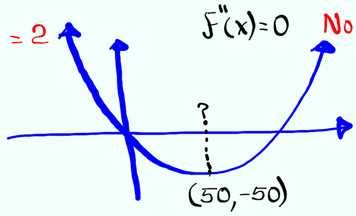
→ $x$ & $y$ $\qquad$ Product $xy$

$x - y = 100$
isolate $y$ $\qquad y = x - 100$

$x(x-100)$
$f(x) = x(x-100)$
$f(x) = x^2 - 100x$

$f'(x) = 2x - 100 \qquad f'(x) = 0 \qquad 2x - 100 = 0 \rightarrow x = 50$

$f''(x) = 2 \qquad f''(x) = 0 \quad$ No




Apr 23-10:47 AM

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(x) = \tan x$

$a = \frac{\pi}{4}$

$f'(x) = \sec^2 x$

Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$

$= f'\left(\frac{\pi}{4}\right) = \sec^2 \frac{\pi}{4} = \left(\sqrt{2}\right)^2 = \boxed{2}$

$f(x) = \frac{\sin^2 x}{1 - \cos^2 x}$ Find $f'(x)$

$f(x) = \frac{\sin^2 x}{\sin^2 x} = 1$ $f'(x) = 0$

Apr 23-10:56 AM